



# Preliminary identification of the observed pulsation modes of ZZ Ceti star KUV 03442+0719



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## HIGHLIGHTS

- We obtained new photometric time series of the ZZ Ceti star KUV 03442+0719.
- We found rotational splits of  $l = 1$  and  $l = 2$  modes in this star.
- The detection of rotational splits helps us to estimate the mean rotation period.
- By using the asymptotic analysis we get preliminary identification of all of the observed periods.

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## ABSTRACT

KUV 03442+0719 is a ZZ Ceti star originally discovered in 2005. We performed observations for it in 2010, 2011 and 2012. From the three years' Fourier transform spectra, a total number of 43 pulsation periods are detected. We found out a set of complete quintuplets, five sets of incomplete quintuplets and two sets of incomplete triplets among these periods. They are interpreted as rotational splits of  $l = 2$  and  $l = 1$  modes. We thus derive a mean rotation period of  $6.71 \pm 0.11$  h from the values of splitting spacing. We perform asymptotic analysis to get preliminary identification of the observed pulsation modes.

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## 1. Introduction

About 98% of all stars (Winget and Kepler, 2008) will become white dwarf stars (WDs, hereinafter) in the final stage of their evolution. WDs provide us important samples for investigating stellar evolution and testing physics under extreme conditions. The majority (about 80%) of WDs have hydrogen-rich atmospheres and are classified as DA type. When the DA WDs evolve and lie in the range of  $12270 \gtrsim T_{\text{eff}} \gtrsim 10850$  K (Castanheira et al., 2007) they become pulsators. They are known as the DAV or ZZ Ceti stars. There are 148 (Castanheira et al., 2010) ZZ Ceti stars having been found so far.

KUV 03442+0719 is a ZZ Ceti star, which was discovered to be a pulsator in 2005 (Gianninas et al., 2006). A dominant period of 1384.9 s was detected in those observations. After that, a series of spectroscopic observations were performed on it (Koester

et al., 2009; Limoges and Bergeron, 2010; Gianninas et al., 2011). But no photometric observations had been carried out since then. We intended to perform new photometric observations for KUV 03442+0719 since 2010. Our goal was to collect more data to make further asteroseismological study for this star.

In this paper, we report preliminary results of the observations of KUV 03442+0719. The observations are described in Section 2. The results are described in Section 3. In Section 3.1 we give the introduction of rotational splits; in Section 3.2 we discuss a set of possible quintuplets; in Section 3.3 we discuss other splitting components; in Section 3.4 we perform mode identification on the splits and in Section 3.5 we perform asymptotic analysis. The conclusions are given in Section 4.

## 2. Observations

KUV 03442+0719 was observed in 2010 November (3 nights), 2011 November (7 nights) and 2012 December (9 nights), respectively. All observations were carried out on the 2.4-m

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telescope located at Lijiang station of Yunnan Observatories, Chinese Academy of Sciences. Two instruments are available for photometry on the telescope: the Yunnan Faint Object Spectrograph and Camera (YFOSC) which has a CCD size of  $2048 \times 2048$  and a field of view (FOV) about  $9.6 \times 9.6$  arcmin<sup>2</sup>, and the Princeton Instruments VersArray:1300B CCD camera (PI camera), which has a CCD size of  $1300 \times 1340$  and a FOV about  $4.5 \times 4.5$  arcmin<sup>2</sup>. The YFOSC was employed in 2010 and 2012, in which the exposure time was uniformly set to 30 s. The PI camera was used in 2011's observations and the exposure time was set as 40 s. Johnson *B* filter was used all the time.

All of the CCD images are reduced with the IRAF routines. We follow a standard procedure of data reduction. Bias and flat are corrected during the pre-reduction. However, the dark correction is ignored since the chip temperature of the two cameras is always  $-120$  °C with liquid nitrogen cooling and the dark current is negligibly low. The IRAF APPHOT package is adopted to perform aperture photometry. By subtracting the magnitude of comparison star from that of the target star, the differential light curves are finally obtained. The time scale of the photometric time series is converted to the Heliocentric Julian Date (HJD) in order to remove the effects of movement of the Earth around the Sun.

The method of Fourier analysis helps us to find the information about pulsation frequencies from the light curves. The software PERIOD04 (Lenz and Breger, 2005), which was designed to analyse multi-periodic astronomical time series, is adopted to perform Fourier transform (FT) on the light curves. The data collected in each year's observations are combined as a dataset. Three FT spectra are obtained and shown in Fig. 1. We extract the pulsation signals with their frequencies, amplitudes and phases from the FT spectra according to a standard pre-whitening procedure. Following a widespread criterion (Breger et al., 1993; Kuschnig et al., 1997), the signals whose level of signal-to-noise ratio (S/N) are greater than 4 are taken as reliable signals. We finally detect 14, 26 and 11 signals from the light curves of 2010, 2011 and 2012, respectively. The signals and their values of frequency, period,

amplitude and S/N are listed in Table 1. The uncertainties of the frequency ( $\sigma_f$ ) are given along with the frequencies, which are estimated with a Monte Carlo Simulation, as those done in the previous work of ours (Fu et al., 2013; Su et al., 2014).

Some signals are found to be close to each other in frequency but detected in different years. If their frequencies overlap within  $f \pm 3\sigma_f$  range, we take them as the same mode. Thus the four pairs of frequencies  $F_{119}$  (808.13  $\mu$ Hz) and  $F_{204}$  (807.76  $\mu$ Hz);  $F_{120}$  (834.47  $\mu$ Hz) and  $F_{205}$  (834.21  $\mu$ Hz);  $F_{010}$  (1003.07  $\mu$ Hz) and  $F_{126}$  (1003.52  $\mu$ Hz) and  $F_{011}$  (1024.56  $\mu$ Hz) and  $F_{210}$  (1023.95  $\mu$ Hz) are considered to be the same.

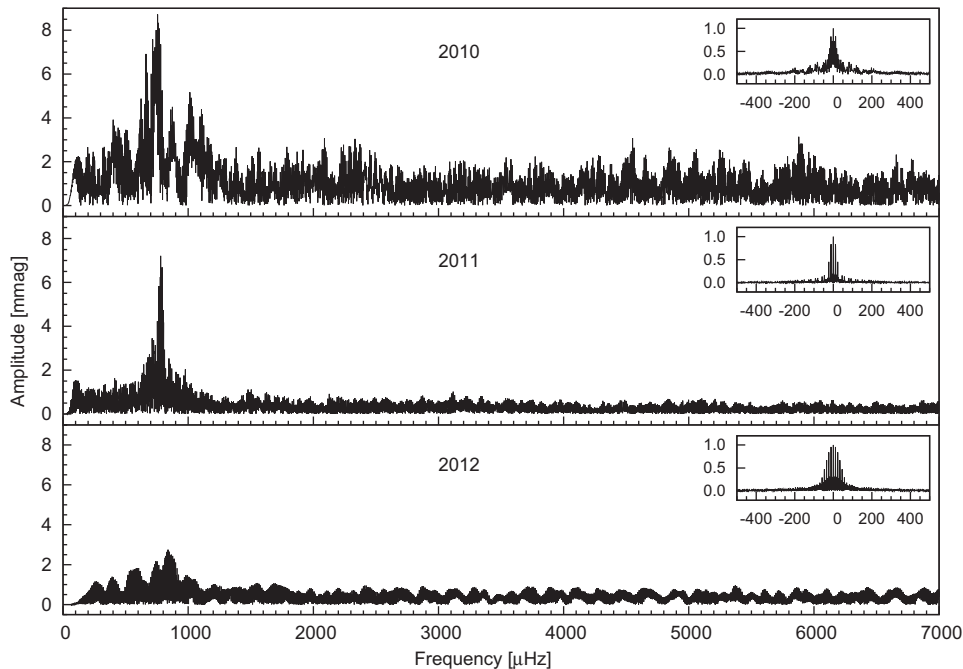
Some of the frequencies are probably linear combinations, which should be found out. For each set of linear combination, we identify higher amplitude ones to be independent and the lowest amplitude one to be the combination. We find the following combinations.

1.  $F_{003}$  (622.07)  $\approx F_{011} - F_{001}$  (1024.56 – 400.83 = 623.73).
2.  $F_{101}$  (102.14)  $\approx F_{117} - F_{107}$  (780.12 – 678.17 = 101.95).
3.  $F_{102}$  (119.49)  $\approx F_{122} - F_{112}$  (856.84 – 736.93 = 119.91).
4.  $F_{103}$  (193.01)  $\approx F_{120} - F_{106}$  (834.47 – 642.54 = 191.93).

### 3. Preliminary results of the observations

#### 3.1. Rotational splits

It is known that pulsations observed in white dwarfs are non-radial oscillations. Each individual oscillation mode is characterized by the spherical harmonic degree  $l$ , the radial order  $n$  and the azimuthal order  $m$ . In the ideal spherically symmetric condition,  $m$  would be degenerate, i.e. modes with the same  $l$  and  $n$  but different  $m$  will have the same frequency. Actually, the existence of rotation breaks the star's spherical symmetry and removes the degeneracy of  $m$ . For high-order  $g$  modes, a second-order expression of Dziembowski and Goode, 1992 showed that



**Fig. 1.** Fourier spectra of light curves observed in 2010, 2011 and 2012, respectively. The frequency is in micro-hertz ( $\mu$ Hz) and the amplitude in milli-magnitude (mmag). All spectra are only showed with frequency range from 0 to 7000  $\mu$ Hz. The insets show the spectral windows.

**Table 1**

The S/N > 4 signals extracted from the FT spectra of 2010, 2011 and 2012. The values of frequency, period, amplitude and S/N are given. In each year, signals are listed according to the frequency values from low to high.

Year	ID	Frequency (μHz)	Period (s)	Amplitude (mmag)	S/N
2010	F <sub>001</sub>	400.83 ± 0.42	2494.83	4.31	5.14
	F <sub>002</sub>	410.00 ± 0.49	2439.00	3.72	4.19
	F <sub>003</sub>	622.07 ± 0.38	1607.53	3.68	4.19
	F <sub>004</sub>	662.79 ± 0.29	1508.77	5.45	6.58
	F <sub>005</sub>	685.93 ± 0.27	1457.87	5.02	6.51
	F <sub>006</sub>	741.17 ± 0.20	1349.22	8.81	10.17
	F <sub>007</sub>	755.67 ± 0.20	1323.32	9.80	11.54
	F <sub>008</sub>	877.50 ± 0.82	1139.60	4.00	4.20
	F <sub>009</sub>	879.90 ± 0.72	1136.49	4.83	5.43
	F <sub>010</sub>	1003.07 ± 0.39	996.94	3.92	4.18
	F <sub>011</sub>	1024.56 ± 0.27	976.03	4.74	5.90
	F <sub>012</sub>	1100.58 ± 0.32	908.61	4.13	4.84
	F <sub>013</sub>	2315.97 ± 0.35	431.79	3.59	4.28
	F <sub>014</sub>	2332.85 ± 0.27	428.66	4.62	5.49
2011	F <sub>101</sub>	102.14 ± 0.16	9790.14	1.72	4.79
	F <sub>102</sub>	119.49 ± 0.18	8368.75	1.58	4.38
	F <sub>103</sub>	193.01 ± 0.19	5181.19	1.48	4.08
	F <sub>104</sub>	412.26 ± 0.14	2425.66	1.61	4.52
	F <sub>105</sub>	632.43 ± 0.20	1581.21	1.58	4.09
	F <sub>106</sub>	642.54 ± 0.25	1556.31	1.56	4.25
	F <sub>107</sub>	678.17 ± 0.12	1474.55	2.40	6.53
	F <sub>108</sub>	702.39 ± 0.33	1423.70	2.48	5.83
	F <sub>109</sub>	712.42 ± 0.21	1403.68	3.39	10.35
	F <sub>110</sub>	716.31 ± 0.31	1396.05	1.67	5.01
	F <sub>111</sub>	729.95 ± 0.20	1369.96	2.51	5.77
	F <sub>112</sub>	736.93 ± 0.11	1356.97	3.87	10.68
	F <sub>113</sub>	743.32 ± 0.28	1345.31	2.35	7.03
	F <sub>114</sub>	746.08 ± 0.33	1340.35	1.71	5.02
	F <sub>115</sub>	775.43 ± 0.37	1289.60	1.40	14.66
	F <sub>116</sub>	777.00 ± 0.28	1287.00	5.53	4.22
	F <sub>117</sub>	780.12 ± 0.05	1281.86	6.72	19.43
	F <sub>118</sub>	799.78 ± 0.06	1250.35	4.95	13.71
F <sub>119</sub>	808.13 ± 0.10	1237.43	2.49	6.86	
F <sub>120</sub>	834.47 ± 0.12	1198.36	2.71	7.11	
F <sub>121</sub>	838.51 ± 0.16	1192.59	1.82	4.89	
F <sub>122</sub>	856.84 ± 0.10	1167.07	2.58	6.70	
F <sub>123</sub>	911.16 ± 0.16	1097.50	1.98	5.69	
F <sub>124</sub>	925.31 ± 0.16	1080.72	2.10	5.92	
F <sub>125</sub>	976.67 ± 0.15	1023.89	1.64	4.77	
F <sub>126</sub>	1003.52 ± 0.16	996.49	1.72	4.81	
2012	F <sub>201</sub>	553.47 ± 0.29	1806.78	1.68	4.67
	F <sub>202</sub>	592.94 ± 0.18	1686.51	2.11	5.65
	F <sub>203</sub>	753.71 ± 0.14	1326.76	2.40	6.95
	F <sub>204</sub>	807.76 ± 0.18	1238.00	2.44	5.46
	F <sub>205</sub>	834.21 ± 0.13	1198.74	2.73	8.07
	F <sub>206</sub>	843.44 ± 0.19	1185.63	2.60	5.55
	F <sub>207</sub>	864.54 ± 0.14	1156.68	2.54	6.63
	F <sub>208</sub>	969.79 ± 0.14	1031.15	2.36	6.08
	F <sub>209</sub>	1021.70 ± 0.26	978.76	1.88	5.13
	F <sub>210</sub>	1023.95 ± 0.29	976.61	1.62	4.20
	F <sub>211</sub>	1027.85 ± 0.27	972.90	2.02	5.25

Substituting these components into Eq. (1), we can derive the rotation period

$$P_{\text{rot}} = \frac{2\pi}{\Omega}. \quad (2)$$

We assume the central frequency 712.42 μHz as  $m = 0$  mode and the frequencies 642.54, 678.17, 746.08 and 780.12 μHz are, respectively of  $m = -2$ ,  $m = -1$ ,  $m = 1$  and  $m = 2$ . We thus get the values of rotation period of 6.75, 6.82, 6.81 and 6.71 h, respectively. It is surprising that the frequency 780.12 μHz, which is the largest amplitude signal, acts as a splitting product of  $m = 2$ , while other frequencies all have lower amplitude than the central frequency.

### 3.3. Other rotational splitting components

In the data of 2010 observations, frequencies 685.93 and 755.67 μHz have a spacing of 69.74 μHz. It is about twice of the mean spacing of the quintuplets. It is reasonable to assume them as two components of the quintuplets. The relation between them is  $\Delta m = 2$ . Note that the frequency 755.67 μHz has the largest amplitude, which is thus assumed as  $m = 0$  mode. The frequency 685.93 μHz is then identified as  $m = -2$  mode. Using the frequency spacing, we derive a rotation period of 6.76 h. This value is coincident with those of the identified quintuplets. In addition, the frequencies 1003.07 and 1024.56 μHz seem to be two components of a set of triplets, which have a frequency spacing of 21.49 μHz. We identify the higher amplitude frequency 1024.56 μHz as  $m = 0$  mode and the other is of  $m = -1$ . A rotation period of 6.46 h is thus derived by the frequency spacing.

Looking back to the 2011's data, there are some frequencies to form four sets of incomplete quintuplets. They are (632.43, 702.39 and 736.93 μHz), (743.32 and 777.00 μHz), (729.95, 799.78 and 834.47 μHz), and (856.84 and 925.31 μHz). For a set that contains three frequencies, the central frequency is assumed to be  $m = 0$  mode and the  $m$  values of other components can be identified distinctly according to the frequency spacing. For a set that only contains two frequencies, the frequency with higher amplitude is assumed to be  $m = 0$  mode and the other one is thus identified by the frequency spacing.

We do not find any quintuplets in the 2012's observations. But two frequencies 843.44 and 864.54 μHz are found to be a possible set of incomplete triplets. We assume the higher amplitude one as  $m = 0$  mode and the other one is  $m = 1$  mode. A rotation period of 6.60 h is derived in terms of the frequency spacing. The value is also coincident with the above-mentioned ones. These rotational splits (quintuplets and triplets) are listed in Table 2, where we can also find their frequencies and the values of frequency spacing,  $m$  and rotation period.

### 3.4. Mode identification

The detection of triplets and quintuplets helps us to identify the  $l$  values of them. As a result, the frequencies 1003.07 and 1024.56 μHz observed in 2010 and the frequencies 843.44 and 864.54 μHz observed in 2012 are identified as  $l = 1$  modes. The frequencies 685.93 and 755.67 μHz observed in 2010 and the frequencies 632.43, 642.54, 678.17, 702.39, 712.42, 729.95, 736.93, 743.32, 746.08, 777.00, 780.12, 799.78, 834.47, 856.84 and 925.31 μHz observed in 2011 are identified as  $l = 2$  modes. The frequencies 1003.52 and 1023.95 μHz can also be identified as  $l = 1$  modes and the frequency 834.21 μHz as  $l = 2$  mode, because they are considered to be the same as 1003.07, 1024.56 and 834.47 μHz (see Section 2). The other frequencies are still unidentified, for the absence of frequency splitting. An asymptotic

$$\omega_{l,n,m} = \omega_{l,n} + m\Omega \left( 1 - \frac{1}{L^2} \right) - \frac{m^2\Omega^2}{\omega_{l,n}} \frac{4L^2(2L^2 - 3) - 9}{2L^4(4L^2 - 3)}, \quad (1)$$

where  $\Omega$  is the rotation frequency and  $L^2 = l(l+1)$ . This expression produces fine-structure, which split a frequency  $\omega_{l,n}$  into  $2l+1$  components ( $m = -l, \dots, 0, \dots, l$ ). Due to the second-order term, the splitting values between different frequencies will not be equal.

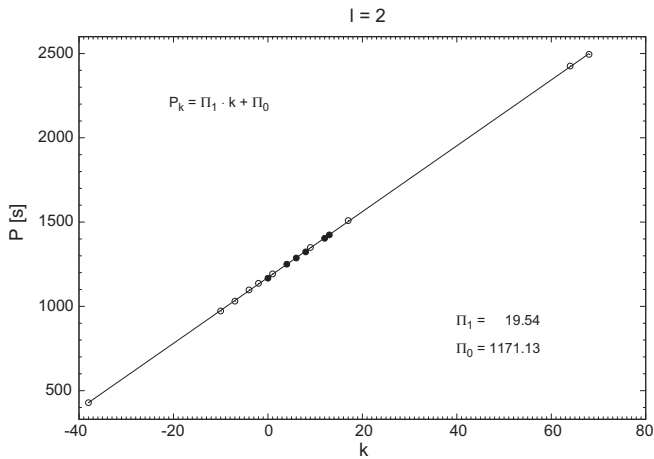
### 3.2. A set of possible quintuplets

We search for potentially rotational splits by subtracting the adjacent frequencies. In the data of 2011's observations, we find five frequencies 642.54, 678.17, 712.42, 746.08 and 780.12 μHz to have approximately equal frequency spacing around 35 μHz. It hence inspires us to identify them as a whole set of quintuplets, which corresponds to the rotational split of an  $l = 2$  mode.

**Table 2**

Possible rotational splits found in observations.  $f$  is frequency,  $\delta f$  is the spacing between frequencies and  $P_{\text{rot}}$  is rotation period derived by the frequency spacing.

Year	$f$ ( $\mu\text{Hz}$ )	$\delta f$ ( $\mu\text{Hz}$ )	$m$	$P_{\text{rot}}$ (h)	Year	$f$ ( $\mu\text{Hz}$ )	$\delta f$ ( $\mu\text{Hz}$ )	$m$	$P_{\text{rot}}$ (h)
Quintuplets					Quintuplets				
2010	685.93		-2		2011	743.32		-1	
		69.74		6.76			33.68		6.93
	755.67		0			777.00		0	
2011	632.43		-2		2011	729.95		-2	
		69.96		6.75			69.83		6.74
	702.39		0			799.78		0	
		34.54		6.64			34.69		6.62
	736.93		+1			834.47		+1	
2011	642.54		-2		2011	856.84		0	
		35.63		6.75			68.47		6.65
	678.17		-1			925.31		+2	
		34.25		6.46	Triplets				
	712.42		0		2010	1003.07		-1	
		33.66		6.81			21.49		6.46
	746.08		+1			1024.56		0	
		34.04		6.71	2012	843.44		0	
	780.12		+2				21.10		6.60
						864.54		+1	



**Fig. 2.** Least square fitting of periods of  $l=2$ ,  $m=0$  modes. The straight line represents the function of Eq. 4 and each point represents an observed period. The solid point denotes the  $m=0$  component of the quintuplets. The integer  $k$  reflects the relative relationship between different radial overtones. The fitting results are shown in the figure.

analysis could provide clues to the identification of those frequencies.

### 3.5. Asymptotic analysis

The asymptotic expression of  $g$  mode (see [Unno et al., 1979](#); [Tassoul, 1980](#) for more details) shows that

$$P_{l,n} \approx \frac{2\pi^2 n}{L} \left( \int_0^R \frac{N}{r} dr \right)^{-1}, \quad (3)$$

where  $P_{l,n}$  is the period of a mode with  $l$  and  $n$  and  $L = \sqrt{l(l+1)}$  as before. For a specified star, the integration is approximately a fixed value. It implies that adjacent radial overtones of the same  $l$  distribute approximately in an equal period spacing. We can thus adopt a linear function

$$P_k = \Pi_1 \cdot k + \Pi_0, \quad (4)$$

to fit the periods, where  $P_k$  is period,  $\Pi_1$  is the mean period spacing and  $\Pi_0$  is a constant concerning the zero point. The integer  $k$  is not

**Table 3**

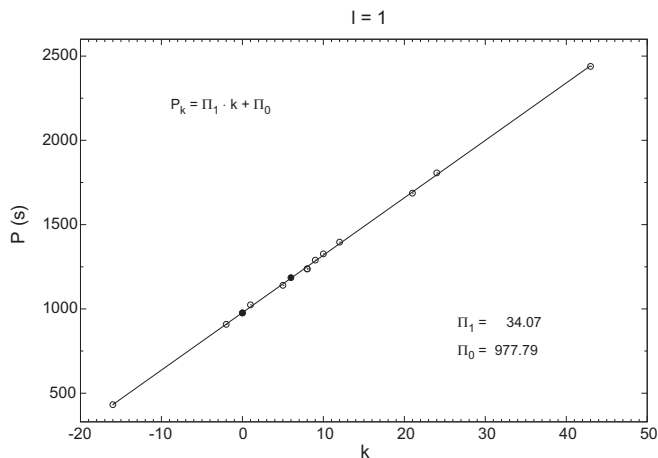
Results of identification of  $l$  by the linear fitting. We list the year of observations, period, frequency and fitted  $k$  value of each  $m=0$  mode.

$l=1$				$l=2$			
Year	Period (s)	Frequency ( $\mu\text{Hz}$ )	$k$	Year	Period (s)	Frequency ( $\mu\text{Hz}$ )	$k$
2010	2439.02	410.00	43	2010	2494.82	400.83	68
2012	1806.78	553.47	24	2011	2425.66	412.26	64
2012	1686.51	592.94	21	2010	1508.77	662.79	17
2011	1396.05	716.31	12	2011	1423.70	702.39	13
2012	1326.76	753.71	10	2011	1403.68	712.42	12
2011	1289.60	775.43	9	2010	1349.22	741.17	9
2012	1238.00	807.76	8	2010	1323.32	755.67	8
2011	1237.43	808.13	8	2011	1287.00	777.00	6
2012	1185.63	843.44	6	2011	1250.35	799.78	4
2010	1139.60	877.50	5	2011	1192.59	838.51	1
2011	1023.89	976.67	1	2011	1167.07	856.84	0
2012	976.61	1023.95	0	2010	1136.49	879.90	-2
2010	976.03	1024.56	0	2011	1097.50	911.16	-4
2010	908.61	1100.58	-2	2012	1031.15	969.79	-7
2010	431.79	2315.97	-16	2012	972.90	1027.85	-10
				2010	428.66	2332.85	-38

equal to the radial order  $n$ , but it reflects the relative relationship between different radial overtones because of  $\Delta k = \Delta n$ .

We perform an initial fitting to those identified  $l=2$  modes, which are in the majority of all identified modes. The six modes of  $l=2$ ,  $m=0$  (i.e. the central components of quintuplets) are chosen to perform the linear fitting. Their periods are 1167.07, 1250.35, 1287.00, 1323.32, 1403.68 and 1423.70 s. We suppose the  $k$  values of them as 0, 4, 6, 8, 12 and 13. Fitting those periods with Eq. 4, we derive that  $\Pi_1 = 19.59 \pm 0.20$  s and  $\Pi_0 = 1168.79 \pm 1.66$  s.

We then search among the unidentified periods by taking the six identified  $l=2$ ,  $m=0$  modes and the mean period spacing as references to find out the periods which satisfy Eq. 4 approximately. We find that the periods 428.66, 972.90, 1031.15, 1097.50, 1136.49, 1192.59, 1349.22, 1508.77, 2425.66 and 2494.82 s could satisfy the asymptotic expression, if their  $k$  values are supposed as -38, -10, -7, -4, -2, 1, 9, 17, 64 and 68, respectively. They are thus identified as  $l=2$  modes. We perform linear fitting again using the periods of these modes as well as the six references. The fitting result is  $\Pi_1 = 19.54 \pm 0.04$  s and



**Fig. 3.** Same as Fig. 2. Least square fitting of periods of  $l = 1$ ,  $m = 0$  modes. The straight line represents the function of Eq. 4 and each point represents a observed period. The solid point denotes the  $m = 0$  component of the triplets. The integer  $k$  reflects the relative relationship between different radial overtones. The fitting results are also shown in the figure.

$\Pi_0 = 1171.13 \pm 0.95$  s. The fitting result is shown in Fig. 2. All of these modes are listed in Table 3.

We then try to fit the rest of unidentified periods with the asymptotic expression of  $l = 1$ . The periods 976.03, 976.61 and 1185.63 s are identified as  $l = 1$  modes in Section 3.3. They can be taken as references. The mean period spacing of  $l = 1$  modes can be derived from the asymptotic expression, which shows that  $\Pi_1 \propto L^{-1}$ . Hence  $\Pi_{1,l=1} \approx \sqrt{3} \cdot \Pi_{1,l=2} = 33.83$  s. We suppose the periods 976.03 and 976.61 s as  $k = 0$ . The period 1185.63 s is of  $k = 6$  according to the mean period spacing. The periods 431.79, 908.61, 1023.89, 1139.60, 1237.43, 1238.00, 1289.60, 1326.76, 1396.05, 1686.51, 1806.78 and 2439.02 s are found to satisfy the asymptotic expression of  $l = 1$  and the corresponding  $k$  values are  $-16, -2, 1, 5, 8, 8, 9, 10, 12, 21, 24$  and 43. Note that, the periods 1237.43 and 1238.00 s are considered the same. These modes are also listed in Table 3. Fitting these periods and the references with Eq. 4, we get a result of  $\Pi_1 = 34.07 \pm 0.17$  s and  $\Pi_0 = 977.79 \pm 2.62$  s. The fitting result is shown in Fig. 3.

There is a period 978.76 s which is neither categorized as  $l = 1$  mode, nor categorized as  $l = 2$ . A possibility is that this period is an unidentified combination whose components are not detected for some reasons. On the other hand, it also inspires us to suppose this period as a mode of  $m \neq 0$ . We firstly suppose it as  $l = 1$ ,  $m \neq 0$  modes. For the case of  $m = -1$ , the central frequency ( $m = 0$ ) is about 1041.70  $\mu\text{Hz}$  according to the the frequency spacing of  $l = 1$  ( $\sim 21$   $\mu\text{Hz}$ , see Table 2) and the corresponding period is about 959.97 s. For the case of  $m = 1$ , the  $m = 0$  component is about 1001.70  $\mu\text{Hz}$  (998.30 s). Neither the period 959.97 s nor the period 998.30 s could satisfy the asymptotic expression of  $l = 1$ . We then consider the cases of  $l = 2$ ,  $m \neq 0$ . The period 978.76 s is then supposed as  $m = -2$ ,  $m = -1$ ,  $m = 1$  and  $m = 2$ , respectively. The corresponding periods of the  $m = 0$  component are about 917.68, 947.24, 1012.45 and 1048.55 s, supposing the frequency spacing is  $\sim 34$   $\mu\text{Hz}$ . We find that the period 917.68 s could satisfy the asymptotic expression of  $l = 2$  with  $k = -13$  (the theoretical value is 917.11 s). Moreover, the period 1012.45 s could also satisfy the asymptotic expression of  $l = 2$  with  $k = -8$  (the theoretical value is 1014.81 s). So it provides possibility to identified the period 978.76 s as an  $l = 2$ ,  $m = -2$  or an  $l = 2$ ,  $m = 1$  splitting component, and the corresponding central frequency is 1089.70 or 987.70  $\mu\text{Hz}$ . However, the identification of the period 978.76 s is still suspicious. We leave the question open at this stage. We

expect to find new evidences in the future to help us with the final confirmation.

#### 4. Conclusions

More observations for the ZZ Ceti star KUV 03442+0719 were performed in 2010, 2011 and 2012, and new photometric time series were obtained. A total number of 51 significant signals have been extracted from the three years' FT spectra. Four of them are identified as linear combinations. In addition, four signals are found to appear in different years' FT spectra. But we have not find any signal to appear in all three years' FT spectra.

An interesting result is that we have found out a set of complete quintuplets among the modes detected in 2011. In addition, other five sets of incomplete quintuplets are also found in the data of 2010 and 2011. These quintuplets are interpreted as frequency splits of  $l = 2$  modes due to rotation. This is the first time that rotational splits of  $l = 2$  modes have been found in this star. It is usually thought that modes with  $l > 1$  are hard to be observed, because of geometrical effect. Our result shows a possibility of observing rotational splits of  $l = 2$  modes. Moreover, two sets of incomplete triplets are found in the data of 2010 and 2012. Using the frequency spacing of these triplets and quintuplets, we estimate a mean rotation period of  $6.71 \pm 0.11$  h.

The detection of rotational splits helps us to do identification with the corresponding period. Using these identified periods as references, we perform asymptotic analysis on the remaining periods. By using the asymptotic analysis we get preliminary identification of all of the observed periods.

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